JV-003-04017001 Seat No. ________

B. Sc. - M. Sc. (Applied Physics) (Sem. VII) (CBCS) Examination

October - 2019

Core-I, Paper-I : Mathematical Methods in Physics

(New Course)

Faculty Code : 003
Subject Code : 04017001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) All questions are compulsory.

(2) Numbers in the right margin indicate marks.

1 Attempt any seven short questions : **14**

(1) Define : Curl and Divergence.

(2) Find out the gradient of below functions.

(i) $\varnothing = x^2 yz^3$,

(ii) $\varnothing = yz + xz + xy$

(3) Find the curl of following :

(i) $\rho \sin\varnothing\, \hat{\rho} + \rho^3 z\, \hat{\varnothing} + z\cos\varnothing\, \hat{z}$

(ii) $\frac{1}{r^2}\sin\theta\, \hat{r} + r\sin\theta\cos\varnothing\, \hat{\theta} + \cos\theta\, \hat{\varnothing}$.

(4) Define analytic function.

(5) To check following functions are analytic or not ?

(i) $f(z) = \cosh z$

(ii) $f(z) = \sin z$.

(6) Write the advantages of fourier series.

(7) Find the fourier series expansion of the pertiodic function of period 2 π, $f(x) = x^2, -\pi \le x \le \pi$.

(8) Define linear differential equation with example.

(9) Write down the Rodrigue's formula and generating function for Legendre's polynomial.

(10) Solve : $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 5e^{3x}$.

2 (a) Write answer of any two : **10**

(1) State and prove Stoke's theorem.

(2) Find out unit normal at $\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}}\right)$ on the surface of $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

(3) Find out the rate of change of a $\varnothing = x^2y + yz$ at $(1,1,-1)$ in the direction of $\hat{i} + 2\hat{j} + 3\hat{k}$.

(4) To find integral $\int \vec{A}\,\vec{dl}$ from $(1,-2,1)$ to $(3,-2,1)$ to $(3,1,1)$ and $(3,1,4)$ where $\vec{A} = \left(2xy + z^3\right)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$.

(b) Write answer of any one : **4**

(1) State and prove Gauss's divergence theorem.

(2) Which of the following vector is parallel to the surface at the point$(2, -2, 3)$ for $x^2 + 2xz = 4$?

(i) $-6\hat{i} - 2\hat{j} + 5\hat{k}$

(ii) $6\hat{i} + 2\hat{j} + 5\hat{k}$

(iii) $6\hat{i} - 2\hat{j} + 5\hat{k}$

(iv) $6\hat{i} - 2\hat{j} - 5\hat{k}$

3 (a) Write answers of any two : **10**

(1) State and prove Cauchy integral theorem.

(2) The harmonic conjugate of $u(x,y)=2x(1-y)$ corresponding to complex analytic function is $\alpha x^2+\beta y+\gamma y^2$. Find α,β and γ.

(3) Let $u(x,y)=x+\frac{1}{2}\left(x^2-y^2\right)$ both real part of analytic function then which of the following is imaginary part.

(4) Evaluate $\int \frac{2z+3}{z}\,dz$, where c is the upper half of the circle, $|Z|=2$, in the clockwise direction.

(b) Write answer of any one : **4**

(1) Find the imaginary part of a given function :

(i) x^2-y^2

(ii) x^3-3x^2y

(2) Find singular point and residues of following :

(i) $\dfrac{z^2}{(z-2)(z-1)^2}$

(ii) $\dfrac{1}{z^2+a^2}$.

4 (a) Write answers of any two : **10**

(1) Find out Fourier series for $f(x)$ if,

$f(x)=\begin{cases}-\pi, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$ and deduce that,

$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots\dots\dots=\frac{\pi^2}{8}$.

(2) A periodic function of period 4 is defined as $f(x)=|x|$, $-\pi<x<0$. Find its Fourier series expansion.

(3) Find out fourier cosine transform for $f(x)=e^{-2x}+4e^{-3x}$.

(4) Find out fourier sine integral for $f(x)=e^{-\beta x}\ (\beta>0)$, hence show that, $\frac{\pi}{2}e^{-\beta x}=\int_0^\infty \frac{\lambda\sin\lambda x}{\beta^2+\lambda^2}\,d\lambda$.

(b) Write answer of any one : **4**

(1) Write any four properties of Fourier transforms.

(2) Represent the following function by a Fourier sine series, $f(t)=\begin{cases} t, & 0<t<\frac{\pi}{2} \\ \frac{\pi}{2}, & \frac{\pi}{2}<t<\pi \end{cases}$.

5 (a) Write answers of any two : **10**

(1) Solve : $x^2 y' - 2xy = \frac{1}{x}$.

(2) Solve : $\sin x \frac{dy}{dx} + 2y = \tan^3 \frac{x}{2}$.

(3) Find the value of following :

(i) $\int_{-\infty}^{+\infty} e^{-x^2} H_2(x) H_3(x)\, dx$

(ii) $\int_{-\infty}^{+\infty} e^{-x^2} \left(H_2(x)\right)^2 dx$

(4) Show that $\int_{-1}^{+1} Pn(x)\, dx = 0, n \neq 0$ and $\int_{-1}^{+1} Pn(x)\, dx = 2, n = 0$.

(b) Write answer of any one : **4**

(1) Find the value of $P_1(x)$ and $P_2(x)$ from Rodrigue's formula.

(2) Solve the differential equation $\frac{d^2x}{dt^2} + \frac{g}{l}x = \frac{g}{l}L$.